

Resonance states in the $^{12}\text{C}+^{12}\text{C}$ modified Morse potential

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Abstract

The resonances in $^{12}\text{C}+^{12}\text{C}$ system described earlier using long range Morse potential determined from resonance data itself, are reexamined in the light of the recent development of the two new methods for identification of resonances in the scattering theory, namely Imaginary Test Potential and Imaginary Phase-shift methods. The high lying resonances are found to be not genuine as pointed out by Kato and Abe, and as such are discarded. This discomfiture is due to the (i) shallow behavior of the Morse potential at the outer edge and (ii) inappropriate insertion of Coulomb tail used in that work. These two deficiencies are now removed in the present study by finding a modified Morse potential with steep rise in the outer edge, and joining smoothly to it a term approximating the Coulomb tail. Calculation of the resonances for this modified Morse potential using the above two methods of identification of resonances in our study, yields more than 25 states with angular momenta $0^+ - 12^+$ in the relevant energy regions. This reaffirms the diatomic-like rotational and vibrational picture of the nuclear molecular resonances in $^{12}\text{C}+^{12}\text{C}$ system proposed earlier, and shows close resemblance with the physics of diatomic molecules, a phenomenon belonging to altogether a different area. It is revealing that the similarity extends right upto the level of potential which is Morse type in both the cases. This study also reveals new features of heavy ion potential.

1. Introduction

Since the discovery of nuclear molecular resonances (NMR) in $^{12}\text{C}+^{12}\text{C}$ in 1960 by Bromley et al [1], and subsequent observations [2] of such states in many combinations of projectile and target mainly in low mass systems, fully satisfactory understanding of the mechanism of this phenomenon has not yet been achieved. Numerous attempts both microscopic and phenomenological have met with limited success [3]. First microscopic description with the usual optical potential [4], could account for few low lying states in $^{12}\text{C} + ^{12}\text{C}$, which is a very small part of the rich spectrum consisting about more than 40 states. Use of similar potential [5] could describe only some high lying states in the double excitation model. The band crossing model [6] which is an extension of the model used in [4], was successful in describing some high spin resonances only. Thus microscopically, a holistic description could not be attained. On the other hand, the resonance data show characteristic rotation-vibration structure reminiscent of the spectrum of a diatomic molecule. Iachello [7] postulated that, like the spectra of diatomic molecule, the phenomena of resonance is governed by the dipole degrees of freedom characterized by the vector \vec{r} , the distance between the centers of the two colliding nuclei and consequently the potential $V(r)$. Since $V(r)$ is not known and difficult to determine, he bypassed this completely in favour of an algebraic approach in which the dynamical symmetry group $U(4)$ [a radius vector plus three Euler angles] was used to derive the expression for the energy eigenvalues in terms of the vibrational quantum number n and angular momentum L as

$$E(n, L) = -D + a(n + 1/2) - b(n + 1/2)^2 + cL(L + 1), \quad (1)$$

where D , a , b and c are parameters to be determined by fitting the spectra. Similar formula was evolved using repulsive Eckart potential by Sahu et al [8]. Erb and Bromley [9] fitted 28 resonances to the above expression quite satisfactorily and obtained the rotational parameter c to be 0.76 MeV which corresponds to an intrinsic dumb-bell configuration consisting of two touching ^{12}C nuclei. This implies an equilibrium separation of approximately 6.75 fermi for the two ^{12}C nuclei. This suggests that if a bonding potential like non-absorptive Morse potential between two ^{12}C nuclei exists, then it should have longer range than the usual optical potential. It conjures a picture of parallelism between the physics of diatomic molecules and NMR — two phenomena belonging to two different areas of physics, namely nuclear physics, and atomic and molecular physics governed entirely by two different types of interaction.

That the similarity could extend right upto the level of potential which is Morse type in both the cases, would be indeed illuminating.

Since the heavy-ion potential calculated in different models have similar features like the Morse potential [10] which has rotation-vibration spectrum, we had represented [11] the effective bonding potential between two ^{12}C ions comprising both the Coulomb and Nuclear parts by a Morse potential plus a constant. Then

$$V_{eff}(r) = A + B[\exp(-2\beta x) - 2\exp(-\beta x)], \quad (2)$$

where, $x = (r - R_0)/R_0$ has four parameters A, B, β, R_0 . The bound-states of this potential are obtained by solving the corresponding Schrödinger equation describing the relative motion of the two ^{12}C ions, which yields eigenvalue spectrum [12] with vibrational quantum number n and angular momentum L as

$$\begin{aligned} E(n, L) = & A - B + \frac{\hbar^2}{2\mu R_0^2} [2\beta\gamma(n + 1/2) - \beta^2(n + 1/2)^2 + L(L + 1) \\ & - 3\frac{(\beta - 1)}{\beta\gamma}(n + 1/2)L(L + 1) \\ & - \frac{9(\beta - 1)^2}{4\beta^4 r^2}(L + 1)^2], \end{aligned} \quad (3)$$

where $\gamma^2 = 2\mu B R_0^2 / \hbar^2$.

We fitted the resonance data with the eigenvalue expression Eq. (3) of the potential and succeeded in obtaining its parameters as $A = 6.99$ MeV, $B = 6.30$ MeV, $\beta = 0.957$ and $R_0 = 6.97$ fm which is represented as the solid curve EFGH in Fig. 1. We had used this potential in our study [3, 13] to find out the resonances in $^{12}\text{C} + ^{12}\text{C}$ system. We had calculated its bound and resonance states and had accounted for more than thirty five states. This potential does not have the required Coulombic behaviour in the outer region, which was neglected in the first instance assuming that it will not have much effect on the bound and resonance states. However, subsequently we have added a Coulomb tail represented by $Z_1 Z_2 e^2 / r$ and recalculated [3, 13] the states with the modified potential and showed that, for low-lying states, the changes in the result are small which increase gradually as one moves to high-lying ones. This trend was very much expected. The results were quite similar to those obtained before the insertion of the Coulomb tail. Thus, about thirty states were well accounted for and the rotational vibrational picture of NMR was reaffirmed. Kato and Abe [14] have reexamined our results. We would like to state that, there are as many as

43 states determined by us which lie inside the pocket of this potential which have been reproduced by them in their calculation [14]. Couple of resonances at the top of the well for each partial wave l are also reproduced. However, the high lying ones calculated by us have been found by them to be not genuine resonances but rather echoes. We do agree that these states are not genuine. In the present paper we are revisiting this problem, finding out the deficiencies and coming out with satisfactory solution. Our effective potential EFGH (Fig. 1), is supposed to contain both the nuclear and Coulomb interaction. This potential is expected to be correct in the interior region upto the Coulomb barrier, beyond which it is not appropriate. When relevant Coulomb potential is subtracted it goes to zero at G corresponding to $r = 15$ fm which has been identified as the position of the Coulomb barrier. Beyond this point the two nuclei are separated and the potential should be Coulombic in nature given by $Z_1 Z_2 e^2 / r$ represented by the dotted curve GI in Fig. 1. We would like to emphasize that our method of determining the potential by fitting Eq. (3) with the resonance data has yielded the Morse potential EFG, valid in the interior region upto the Coulomb barrier G. The Coulomb tail GI is the well known exterior part of this potential. The main features of this Morse potential are a long range of 15 fm, depth of 3.45 MeV and repulsive core of 17.45 MeV.

We observe that our Morse potential has unphysical behavior being highly shallow at the outer edge. To be realistic, it must be steep in that region like the Woods-Saxon potential. The Coulomb tail of the potential must be smoothly connected without any kink at the Coulomb barrier which was not the case in the former study [3, 13] where Coulomb tail GI was inserted in *ad hoc* manner (Fig. 1), as pointed out in [14]. In the present paper, we repair these two deficiencies by finding a potential which retains all the important features of our earlier Morse potential like its long-range, depth, the repulsive soft core, and newly added features of steep rise near the outer edge, and further a long range decreasing tail which simulate the Coulomb interaction, in a natural way as a continuation. We call this potential as modified Morse potential. To calculate the resonances of this potential in a consistent and accurate manner, we have used our two methods specially developed for this purpose [15], namely the Imaginary Test Potential (ITP) and Imaginary Phase Shift (IPS) methods which have been shown to be very reliable in identifying the resonances. We show that around 30 resonance states with $L = 0^+ - 12^+$ are reproduced in the relevant energy regions observed in experiment, thus reaffirming the diatomic-like rotation-vibration structure of NMR.

In Sec. 2, we present the construction of the modified Morse potential from the original Morse potential. Sec.3 describes our method of calculation of resonances in S-Matrix approach using the ITP and IPS methods. In Sec.4, the details of the calculation and the results are presented. Secs. 5 and 6 respectively give the discussion on the results, and the conclusions respectively.

2. Modified Morse Potential

In this section we would like to modify the effective Morse potential derived above to make it more realistic. A major deficiency of this potential is its extreme shallowness in the region just to the left of the barrier position G. Our experience on nuclear potential used in different areas of nuclear physics over the years, has shown that, the potential has to be sharply rising in this region, quite akin to Woods-Saxon form with a small diffuseness. Therefore, for the potential to be realistic it should steeply fall to the left of the point G and smoothly connect to the Coulomb-like repulsive tail on the right. We have constructed such a potential which retains all the essential features like depth, range and repulsive core of our effective Morse potential EFGI, and additionally incorporate the above two features. This potential termed as modified Morse potential will be used in our calculation of resonances which is given by

$$\begin{aligned} V(r) &= V_I e^{-a_I r} + V_0 [\xi_1 - (\xi_1 - \xi_2) \rho_1(r)], \quad \text{if } r \leq R_0 \\ &= V_0 \xi_2 \rho_2(r), \quad \text{if } r > R_0 \end{aligned} \quad (4)$$

where

$$\rho_n(r) = [\cosh^2 \frac{R_0 - r}{d_n}]^{-1}, n = 1, 2 \quad (5)$$

and $V_0 > 0$, has eight parameters $V_0, \xi_1, \xi_2, R_0, d_1, d_2, V_I, a_I$ and quite flexible to admit varieties of shape.

The modified Morse potential, thus generated, is presented in Fig.2(b) with the addition of centrifugal term for the angular momenta $l=0^+ - 12^+$. The parameters are $R_0=15$ fm, $d_1 = 0.25$, $d_2 = 15$, $V_0 = 1$ MeV, $\xi_1 = 0.45$, $\xi_2 = 3.45$, $V_I = 17$ MeV and $a_I = 0.57$ fm⁻¹. This potential for $l = 0$ has range 15 fermi, depth 3.45 MeV, and soft core of 17.45 MeV as in original Morse potential. For comparison, in Fig. 2(a), this potential is shown by a solid curve and the original effective Morse potential V_{eff}

for $l = 0$ with the added Coulomb tail shown by a dashed curve. Thus the Coulomb tail in the modified Morse potential is only approximate.

3. Methods of Calculation

To calculate the resonances, we follow the more versatile S-matrix approach [16, 17, 18] rather than the phase-shift method followed earlier in Refs. [3, 11, 13]. The partial wave S-matrix in potential scattering is expressed as

$$S_l(k) = e^{2i\delta_l(k)} = \frac{W[\phi_l(k, r), f_l(k, r)]e^{i\pi l}}{W[\phi_l(k, r), f_l(-k, r)]}, \quad (6)$$

where $W[\phi_l(k, r), f_l(\pm k, r)] = f_l(\pm k)$ are known as Jost functions and W is the Wronskian of the regular $\phi_l(r)$ and the irregular $f_l(\pm k, r)$ solution of the modified Schrodinger equation describing the scattering of two colliding particles with reduced mass μ , center of mass energy E , wave number $k = \sqrt{\frac{2\mu E}{\hbar^2}}$ and phase shift $\delta_l(k)$. The resonances are identified with the poles of $S_l(k)$ arising from the zero (k_p) of $f_l(-k)$ in the fourth quadrant of the complex k -plane such that $k_p = k_r - ik_i$. The resonance energy is

$$E_R = \frac{\hbar^2}{2\mu}(k_r^2 - k_i^2), \quad (7)$$

and the corresponding width is

$$\Gamma = \frac{\hbar^2}{2\mu}4k_r k_i. \quad (8)$$

However in practice, the computation of poles is a non-trivial task, because in the Newton-Raphson type of iteration search programme generally followed, if the starting trial value of k^2 or k is not reasonably close to the resonant pole, the iteration procedure may not lead to exact pole position. So in such procedure, one is not sure if one has succeeded in identifying all the possible resonances present in the system. For large width resonances, the number of iteration for convergence may be unusually large. For very narrow resonances, the chances of skipping the poles in the iteration procedure are quite high. Similar difficulties may arise in the alternative complex scaling method [19, 20]. In fact, a recent study by Sahu *et al* [21] has shown that nearly half of the resonances of Morse potential of Satpathy *et al* calculated by Kato and Abe [14], using complex scaling method are not genuine. To circumvent those difficulties, we have recently developed two methods for unambiguous and authentic identification of resonances in scattering theory, which we have employed in the

present study on $^{12}\text{C} + ^{12}\text{C}$ problem. These are presented in details in Ref. [15]. For easy reading and completeness we summarize them below.

Our methods are based on the fact that, confirmation of resonances in collision experiments are ascertained by their correlated manifestation in different available non-elastic and reaction channels. We use this concept in the theoretical search of resonances of a real potential, by adding a small test imaginary potential (*TIP*) to the real potential under investigation, thereby converting the elastic potential scattering problem, to a two-channel problem involving elastic and absorption channels. Then in the study of the variation of the partial wave reaction cross-section with energy, the position of peaks will be identified as the resonance energy. The method has been termed as Imaginary Test Potential (*ITP*) method. This approach eliminates the difficulties arising from background phase-shift. The resonance positions can be identified remarkably well in this method, since in the calculation one can control the height of the absorption peak in the reaction cross-section by changing the value of the Test Imaginary Potential. The weak resonances can be located easily in this method also.

The second method which is more versatile in the sense that, it gives both the energy and the width of the resonances, has been termed as Imaginary Phase Shift (IPS) method. Here we have made use of the concept of time delay given by $\tau = \frac{1}{k} \frac{d\delta}{dk}$ (in k^{-2} unit) arising due to the trapping of the incident wave in the resonance configuration, normally used in the study of resonance phenomena. By the introduction of TIP, with a strength W , the phase shift becomes complex $\delta = \delta_r + i\delta_i$ resulting in the time-delay

$$\begin{aligned} \tau &= \frac{1}{k} \frac{d\delta}{dk} \\ &= \frac{1}{k} \frac{d\delta_r}{dk} + \frac{i}{k} \frac{d\delta_i}{dk} \\ &= \tau_r + i\tau_i. \end{aligned} \tag{9}$$

The quantity $\tau_r = \frac{1}{k} \frac{d\delta_r}{dk}$ has the usual meaning of time-delay which can be used to estimate the energy of resonance as it has maximum value at resonance energy. This method which has been normally used, has inherent uncertainty due to the requirement of high resolution of energy needed for differentiation. However, now the imaginary part of the time delay τ_i has been shown to have interesting physical property capable of giving both the energy E_R and the width $\frac{\Gamma}{2}$ of resonance. We

have shown [15] that resonance energy E_R will be found by the calculation.

$$\frac{d\delta_i}{dE} \big|_{E=E_R} = 0, \quad \frac{d^2\delta_i}{dE^2} \big|_{E=E_R} < 0, \quad (10)$$

and the width $\Gamma/2$ by

$$\frac{d\delta_i}{dW} \big|_{W=\Gamma/2} = 0, \quad \frac{d^2\delta_i}{dW^2} \big|_{W=\Gamma/2} < 0. \quad (11)$$

Thus IPS method is quite powerful to yield well defined values for the energy of the resonances and their width. In the calculation, the method is found to be quite easy and reliable, since one has to study only the variation of the derivative of δ_i as a function of E and/or W , which is substantially simpler and shorter than the pole search method or complex scaling method, requiring iterative search for resonance parameters with the potential for accumulation of numerical errors.

Although the above two methods are quite satisfactory for large class of potentials one normally encounters in physical problems, there are limitations in treating specific cases. The ITP method is quite fine for identifying the resonances particularly around the barrier top. For the resonance deep inside the pocket, some numerical difficulties can arise if the barrier has large height and width. This is because, in such a case, particularly for lower energy resonances, the wave function will be highly attenuated in the interior region, and hence the reaction cross-section may not show appreciable peak. In such cases, resonance can be located by WKB technique used for bound states.

In the IPS method, the calculation of resonance positions is likely to have same type of problems as stated above since it is equivalent to ITP method. As for the estimation of width, the method works quite well for sharp resonances, because in such cases, the effect of background term is small. The formula (Eq.10-11) is derived using one level Breit-Wigner expression for resonance amplitude which is only an approximation. If the width of the resonance is large, its estimation by this method will not be very satisfactory because of significant background contribution to partial wave amplitude.

4. Calculation of Resonances

As discussed above, for the identification of resonances we need a test imaginary potential V_T which we have chosen as

$$V_T = -iW_0[1 - \rho_1(r)], \quad \text{if } r \leq R_0 \quad (12)$$

with ρ_1 as defined in Eq. (5). We have taken $W_0 = 0.1$ MeV. For different partial waves, we obtain the S-matrix and phase shifts numerically as function of energy.

Following ITP method, we would like to show first that genuine resonance states exists at high energy much above the Coulomb barrier in our potential. For $l=4$ and $l=10$, we calculate all the resonances in both the methods. In Fig. 3(a) and 3(b), $\frac{d\delta_i}{dE}$ and σ_R are plotted as function of E for $l=4$ and in Fig. 4(a) and 4(b), the same quantities for $l=10$. It can be seen that the zero crossing points with negative slopes marked by arrows in Fig. 3(a) and 4(a) indicating the positions of the resonances, perfectly correlate with the peaks in the corresponding plot of reaction cross section σ_R as a function of E in Fig. 3(b) and 4(b), respectively. For $l=4$, there are 5 observable resonances at energies 2.46, 3.67, 5.28, 7.29 and 9.6 MeV. The low lying resonance at 2.46 MeV is not manifested in Fig. 3(a) and 3(b). However, two of these resonances at 7.29 MeV and 9.6 MeV are found to be in the energy region above the usual Coulomb barrier which is about 7 MeV. It must be stated that, the Coulomb barrier of our modified Morse potential is 3.45 MeV. As will be discussed *a posteriori*, this potential is generated in the final phase of the reaction in the exit channel where highly deformed prolate configuration manifests giving rise to lower barrier. The genuineness of these resonances are further ascertained by plotting the wave function $|\psi|$ as function of r in Figs. 5(a) and 5(b) which show the concentration of the wave function in the interior region of the potential. Similarly for $l=10$, the two high lying states at 7.515 and 10.11 MeV, the plot of $|\psi|$ in Figs. 6(a) and 6(b) do confirm the genuineness of the resonances.

It must be emphasized that, the high lying broad resonances above the Coulomb barrier which were absent in our effective Morse potential, occur here, because of the steep rise of the potential at the outer edge giving rise to relative accumulation of wave function in the interior of the potential.

Our search for resonances of the modified Morse potential yielded 37 states with angular momenta $L = 0^+ - 12^+$ which are presented as solid lines in Fig. 7. The circles in the figure represent the experiment. It is satisfying to find that many of our calculated resonances lie in the same energy regions as the experimental spectra. Although quantitative agreement is lacking, the overall picture emerging from our study can be considered to be well supported by experiment. This picture described by our modified Morse potential is entirely due to a real potential. In the actual collision phenomenon, there must be some associated absorptive processes however weak they may be. To simulate this physical feature in our study, we take additionally the

weak imaginary potential $W_0=0.5$ MeV and recalculate the resonances. The results of such calculation are compared with experiment in Fig. 8. It can be seen that 9 low lying states with $l = 0 - 12$ have become extinct giving rise to much better agreement with data. In the calculated spectra for all the five angular momenta $l=0^+, 2^+, 4^+, 6^+, 8^+$, the agreement is reasonable. Thus we can conclude that as many as 25 states predicted with our modified Morse potential are relevant and genuine. Similar results have been obtained in [22] using a potential with a well followed by a wall-like thick barrier of about 7.7 fm. However, how such a heavy-ion potential can be generated using realistic nucleon-nucleon potential, has to be seen.

5. Discussion

The complexity involved in the description of heavy-ion collision has been highlighted here. However, like any other potential scattering approach, the present approach has attempted to by-pass these difficulties, rather overcome it. The two methods ITP and IPS developed by us before [15], and used here in the calculation have yielded unambiguous and authentic results in terms of both the energy and width of the resonances. The energy calculated by both these methods exactly match. Thus, the uncertainty usually inherent in the identification of resonances, and the possibility of missing some of them, are fully eliminated. Therefore, we have complete trust on these resonances, and regard them as the genuine ones of the potential we have used. The resonances thus generated are qualitative in nature confirming the experimental trend which is satisfying — in view of no adjustment of parameters to fit the calculated resonances with the data. It is all the more pleasing that the result is consistent in the sense that the potential determined from the resonance data using the eigenvalues of the Schrodinger equation, describes the resonances calculated in the rigorous S-matrix method. This is indeed parallel to the scheme followed in the study of nuclear spectra. Therefore, considering the goodness of the identification and the moderate quality of the results, we feel a reasonable description of the resonances of $^{12}\text{C}+^{12}\text{C}$ system has been possible. The physics implication of this with regard to our potential, and the resulting mechanism of NMR is discussed below.

(a) Potential

True understanding of the mechanism of nuclear molecular resonances can be feasible, only when we have the proper bonding potential between the two colliding

nuclei derived quantum mechanically based on sound physical principles . Starting with nucleon-nucleon potential determined from the experimental phase-shift analysis, one can fold the density of the colliding nuclei and obtain the heavy-ion potential. The folding potentials thus obtained, have been widely used in heavy-ion physics over the years. The main discomfiture of such potentials is the density profiles of the colliding nuclei — which forms an important element in the calculation — goes on changing in the course of collision. Further it is an evolving entity governed by the reaction dynamics and the bombarding energy. The sudden potential derived using frozen density approximation, may be too simplistic to describe a highly complex phenomenon like NMR. The adiabatic potential pertains to another extreme case which is applicable to slow process involving low bombarding energy. To overcome this difficulty, we had followed a pedagogical quantum mechanical method to determine the potential from the resonance data itself. Treating the nuclei as a two-body problem with a potential acting between them, it was shown that a combination of Morse potential plus a constant is a good representation of the effective potential in the interior region. The corresponding Schrodinger equation has analytic solution which yields an eigenvalue spectrum with rotation-vibration feature. Fitting the resonances with the eigenvalue expression, the parameters of the bonding potential could be determined. The problem is quite similar to the two-body deuteron problem where early informations on n-p potential were derived by supposing a finite square-well represents the essential features of the potential namely, the range and depth. It must be recognized that, because of the amenability of the Schrodinger Equation for Morse potential to analytic solution, we were successful in unearthing the underlying potential. Notwithstanding its unrealistic behavior at the outer edge , its main features like depth, range, and repulsive soft core have been obtained. As shown in the previous section, retaining these features intact, and repairing the outer edge by making it more steep, and incorporating effective Coulomb-like tail, the resonances in $^{12}\text{C} + ^{12}\text{C}$ have been described. All the states have been described as resonances, unlike our previous description [3], where the states lying in the potential well were treated as bound-states, and the states above as resonances. This potential is called modified Morse as it retained all the essential features of general Morse potential in the interior region and only the edge was modified by making it more stiff. It is quite parallel to our traditional use of harmonic oscillator potential — because of its analytic solvability — in our nuclear structure physics, with its unrealistic shallow outer edge. It is usually replaced in special cases by Woods-Saxon potential which

steeply rises in that region.

It may be recalled here that the diatomic molecules show prominent features of rotation and vibration in their spectra which have been well accounted for since 1929, by the use of Morse potential. The present study with a modified Morse potential shows that NMR which are sometimes referred to as nuclear molecules are governed by similar physics as the diatomic molecules. While the former are bound states of two atoms (like O_2), the latter are quasi-bound states of two nuclei, both showing rotation-vibration features governed by Morse type of potential. It is indeed gratifying to see the close resemblance of two different phenomena pertaining to two different areas of physics, namely the molecular physics and nuclear physics, governed by electromagnetic and strong interactions respectively. The present work shows that the similarity extends right upto the level of potential which is Morse type in both the cases. This is indeed quite revealing and satisfying.

(b) Mechanism of resonances

The modified Morse potential determined above, offers a wholesome understanding of the mechanism of NMR. The potential being throughout positive for all values of the coordinate r , will support states which are transient in nature having short life-time like the resonances. Its long-range feature of 15 fm is compatible with the well-known 3α linear chain structure [23] of the first 0^+ excited state of ^{12}C , since two such nuclei aligned along the symmetry axis can generate such a potential. In fact microscopic calculations [3, 24, 25] of ^{12}C - ^{12}C potential — in their highly deformed 3α linear chain configuration — in α -particle, α -cluster and folding models using Brink-Boeker, Volkov and Ali-Bodmore potentials have amply justified this feature. The very fact that the cross sections of $^{12}C+^{12}C$ reaction do not conform exclusively to the picture of two 3α chains colliding in the scattering, one cannot preclude the possibility of the two ^{12}C nuclei can be resonating within a long-range Morse-like potential in the exit channel. The long-range of the potential offer the possibility of the two nuclei to remain all along in the dinuclear regime and interact retaining their identity. This gives a natural explanation for the puzzle to understand how these states which lie at more than 20 MeV of excitation in the corresponding compound system ^{24}Mg without being washed away in the sea of high level density. This discomfiture was temporarily overcome in the past by proposing the hypothesis [26, 27] of the existence of a molecular window with a narrow band of low level density in

the compound nucleus. In the present model, such a postulate is not necessary as the two nuclei retain their identity all along, although individually they are likely to get excited to low-lying prolate-deformed states. In particular in $^{12}\text{C} + ^{12}\text{C}$ system, the 0^+ state at 7.65 MeV in ^{12}C which is a linear chain of three α -clusters, has been supposed by Feshbach [28] to be excited in the NMR phenomena. The level density at such excitation is only few levels per MeV.

The most intriguing feature of the resonance spectrum in a system is that it is not generally random in nature. It shows the characteristics of rotation-vibration similar to the picture seen in a diatomic molecule. This suggests that the genesis of these states must belong to a common substratum conducive for the manifestation of such regularity. The long-range potential is such a substratum which can accommodate spatially the colliding pair of nuclei for adequate time without merging through fusion reaction, and thereby favouring the generation of rotation- vibration spectrum. Thus the natural explanation [3, 11] of the mechanism of NMR proposed earlier, and is strongly endorsed by the present study, is the following.

In the entrance channel, the two spherical nuclei approach each other and develop oblate deformation because of Coulomb interaction. As they come closer either by sub-barrier tunnelling or overcoming the Coulomb barrier, depending upon their energies, they interact strongly as a composite system without losing their identity. In the effort to separate they develop strong prolate deformation in the exit channel which gives rise to a thick Coulomb barrier which inhibits separation. Hence they undergo rotational and vibrational motion like a diatomic molecule and generate NMR. Finally they separate with the restoration of original shape in the exit channel. This feature of collision dynamics has been demonstrated numerically in the surface friction model of Gross and Satpathy [29, 30, 31] before. This long-range potential does not resemble in any way to the sudden or adiabatic potential normally used in various studies. It is visualized to be produced in the final phase of the collision in the exit channel where the system spends relatively longer time in a strongly elongated prolate configuration. This may be considered an effective optical potential in which most of the other channels have been taken into account except the fusion and transfer channel. This potential is the result of the dynamics of the colliding nuclei and therefore the model has been termed earlier as Dynamic Potential Model [3].

6. Conclusion

In summary, we have repaired the two deficiencies of the Morse potential determined earlier from the resonance data of $^{12}\text{C} + ^{12}\text{C}$, namely, the shallow behavior of the potential at the outer edge and the ad-hoc insertion of the Coulomb potential. We have constructed a modified Morse potential which retains all the main features of the old Morse potential like the long-range of 15 fermi, shallow depth and repulsive soft core, and added to it, the new features of steep outer edge and a smoothly continued approximate Coulomb tail. The resonances of this potential have been calculated in the S-matrix approach, using our two newly developed methods namely, Imaginary Test Potential and Imaginary Phase Shift methods. Both the energy and width of above 25 resonance states have been determined in our study in a convincing and authentic way. The calculated states with well defined spin and parity lie in the same energy region where such states are observed in experiment.

Experimentally more than 40 resonances have been observed, some of which lie at very high excitation around 20 MeV. The highest state with $L = 12^+$ predicted by our calculation at 15.3 MeV, is somewhat close to the experiment. The few states above it are probably produced by some other mechanism and cannot be accounted for by our modified Morse potential. Nevertheless, since as many as 25 states have been reasonably well described in our study, it is fair to conclude that modified Morse potential is appropriate for a comprehensive explanation of the resonances in $^{12}\text{C} + ^{12}\text{C}$ system. More importantly it reaffirms the diatomic-like rotation-vibration picture of NMR as concluded before. It is indeed satisfying to note that the present study shows a close resemblance between the physics of diatomic molecules and nuclear molecular resonances, though they belong to different areas governed by different interactions at microscopic level. This resemblance becomes all the more striking as it extends to the level of interaction potential being Morse type in both the cases.

The mechanism of NMR that has naturally emerged is that, the two colliding nuclei in their spherical ground state approach one another in the entrance channel and develop oblate deformation. As they reach the closest proximity either by overcoming the Coulomb barrier or sub-barrier tunnelling, a composite system is formed with individual nucleus retaining its identity. In the exit channel, they develop strong prolate deformation giving rise to a wide Coulomb barrier. Being caught behind the barrier they undergo rotation and vibration in their effort for re-separation and thus generate NMR. Finally they are separated with the restoration of original shape. Thus these states are produced in the final phase of the reaction in the exit channel. This mechanism is distinctly different from other mechanisms in which these

states are produced in the early phase of the collision in the entrance channel before undergoing fusion and other absorptive processes. In the present picture, the entire collision process takes place in the dinuclear regime. The present potential is not ad-hoc unlike in many studies. Its main features are determined from the resonance data itself through a well defined quantum mechanical procedure very much like n-p potential from the deuteron data. The present rigorous calculation of resonances using the same, and their agreement with as many as 25 states, resulting in the plausible explanation of the mechanism shows the study is consistent and theoretically well founded, which most importantly also reveals new features of heavy-ion potential.

Acknowledgements

I would like to express gratitude and appreciation to C.S. Shastry and B. Sahu for extensive discussions, calculation and critical reading of the manuscript. The work was done under the financial support of CSIR, Govt. of India.

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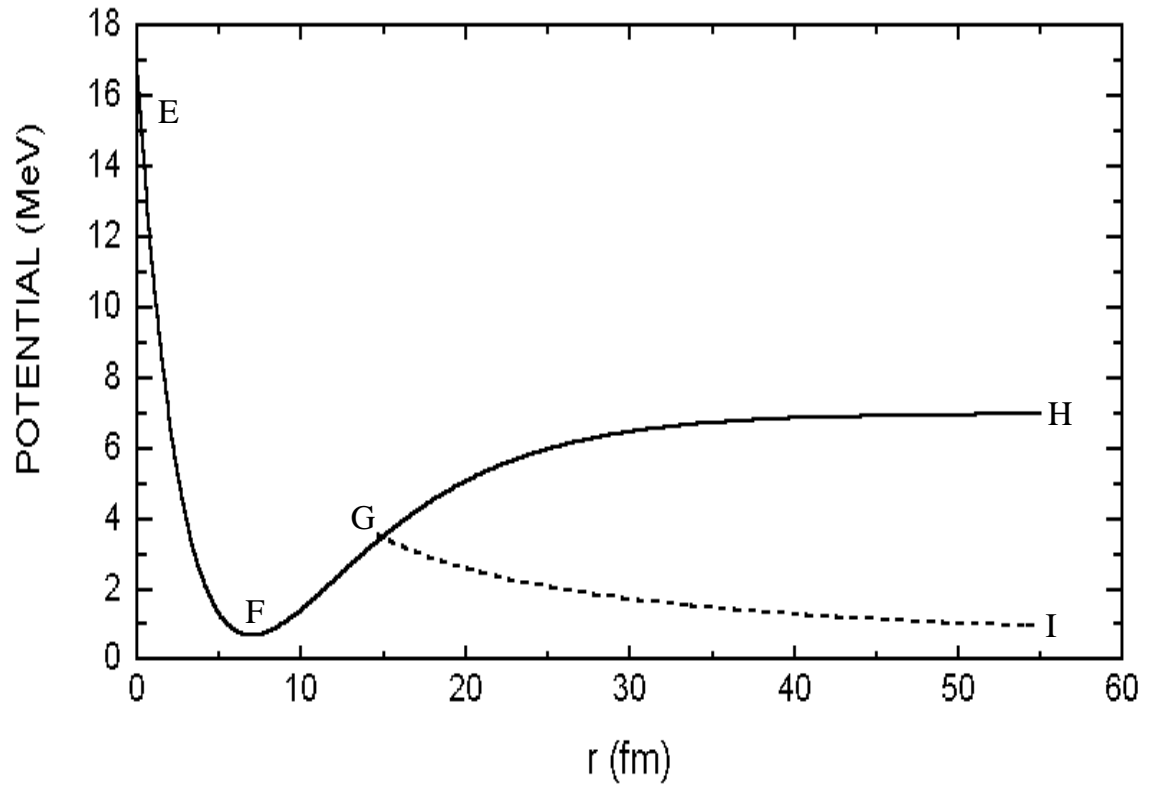


Figure 1: Plot of Morse potential (solid curve) expressed by Eq. (2) as a function of radial position for partial wave $\ell=0$. The dashed curve represents the Coulomb potential for the $^{12}\text{C}+^{12}\text{C}$ system for $r \geq 15$ fm.

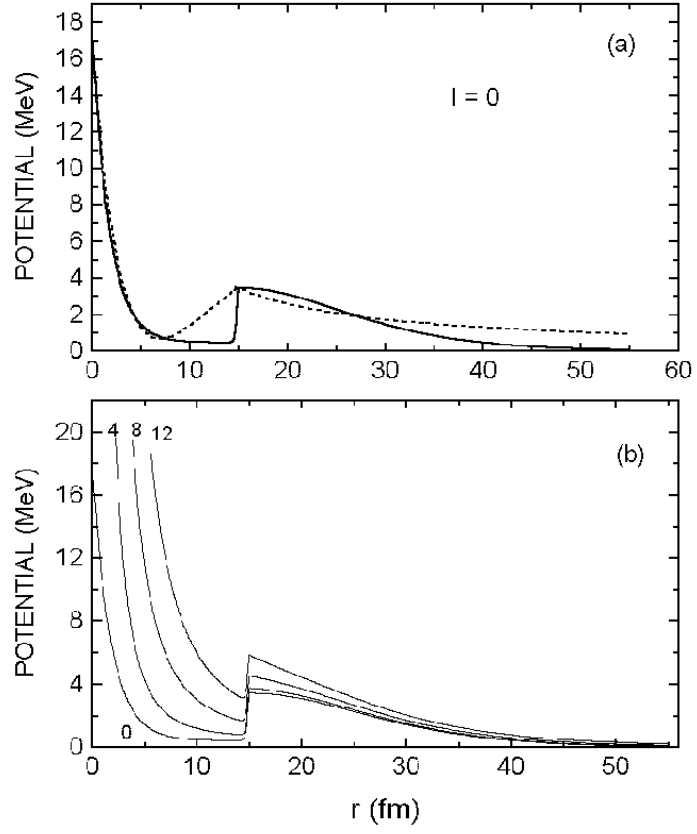


Figure 2: Plot of potential as a function of radial position. (a) Morse potential expressed by (2) with a Coulomb tail beyond $r=15$ fm is compared with newly constructed potential expressed by Eq. (4) for $\ell=0$. (b) The Morse-like potential (4) for different ℓ s = 0, 4, 8, 12.

